Linear Programming

Finite Math

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General Description of Linear Programming

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where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities.

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where a and b are not both zero and the *decision variables* x and y are subject to *constraints* given by linear inequalities.Additionally, x and y must be nonnegative, i.e., $x \geq 0$ and $y \geq 0$.

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(C) *If the feasible region is empty, then both the maximum value and the minimum value of the objective function do not exist.*

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4. *For an applied problem, interpret the optimal solution(s) in terms of the original problem.*

Linear Programming Example

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Maximize and minimize $z = 3x + y$ *subject to the inequalities*

$$\begin{aligned} 2x + y &\leq 20 \\ 10x + y &\geq 36 \\ 2x + 5y &\geq 36 \\ x, y &\geq 0 \end{aligned}$$